Sets (Class-11th maths)
chapter- 1

Set: a collection of well defined objects.

For example:

Set	not Set (not well defined Criteria)
✓ collection of rivers of india	→ collection of large river ×
✓ ——— vowels of english alphabet	→ collection of good × students in class 11th
✓ collection of students who got ~~mark~~ marks above 90%	

Curly braces

Set of vowels of english alphabet $= A = \{a, e, i, o, u\}$

Name of set (Generally capital letter)

objects / members / elements of sets. (Generally in small letter)

Famous Sets:

their Name

Set of natural numbers = N = $\{1, 2, 3, \ldots\}$

Curly Braces

Set of whole numbers = W = $\{0, 1, 2, 3, 4, \ldots\}$

Set of integers = $\underbrace{I = Z}$ = $\{\ldots -3, -2, -1, 0, 1, 2, \ldots\}$

इनमें से कोई सा भी आ सकता है।

Set of rational numbers = Q = $\{-2, 0, 3, \frac{7}{5}, \ldots\}$

rational numbers = $\frac{P}{q}$ ($P, q \Rightarrow$ integers) $q \neq 0$

Set of irrational numbers = $Q' = T$

$= \{\sqrt{2}, \sqrt{3}, \pi, e, 2^{\frac{1}{3}} \ldots\}$ etc.

their decimal expansion has non terminating non repeating pattern

Set of real numbers = R = it has both rational as well as irrational numbers.

Set of positive integers = Z^+ या I^+
$= \{1, 2, 3, ----\}$

Set of positive real numbers = R^+

R_0 = Set of real number except zero.

I_0 या Z_0 = Set of integers except zero
$= \{--- -3, -2, -1, 1, 2, 3, 4, --\}$

Useful symbols: Let $A = \{a, e, i, o, u\}$

Symbols	meaning	
$\in$	$\boxed{\text{belongs to}}$	for example e belongs to A $\Rightarrow e \in A$ Also $i \in A$, $o \in A$, $u \in A$, $a \in A$
$\notin$	$\boxed{\text{doesnot belong to}}$	for example $b \notin A$, $x \notin A$

$\exists$	there exists	we will use these symbols later.
: or |	Such that	
$\forall$	for all values of	

Representation of a Set:

(i) Roster (Tabular) Form
(ii) Set Builder (Property) Form

(in both forms we use curly braces)

(i) Roster Form:

write all the elements in curly braces (no need to repeat the elements & order does not matter)

For example:

✓ Set of natural numbers less than 5

$B = \{1, 2, 3, 4\} = \{2, 1, 3, 4\} = \{4, 3, 1, 2\}$

↑ name of set

all are same

But $B = \{1, 2, \underbrace{3, 3}_{}, \underbrace{4, 4, 4}_{}\}$ repitition

So repitition does not make sense.

✓ Set of letters of word "MATTER"

$= \{M, A, T, E, R\}$

(do not repeat 'T' in set)

(ii) Set Builder form

We do not write each element in this form, we write only property. By reading this property, we can find elements.

such that

$A = \{x : \text{Property of } x\}$

name of set

We read the above expression as — " A is the set of all values of x such that x follows a property"

For example:

Roster Form

$A = \{1, 2\}$

Set builder form

$A = \{x : x \text{ is natural no. less than } 3\}$

$A = \{x : x \text{ is the root of } x^2 - 3x + 2 = 0\}$

$A = \{x : x \text{ is positive factor of } 2\}$

All are Same

Page No.

By the above example we can say that A set can be represented in set builder form in many ways but ~~all~~ all ways gives same set.

we use 'x' for support, in actual 'x' is not an element of set ~~but~~ value of 'x' is an element of set.

eg. Convert $A = \underbrace{\{x : x = \frac{n}{n+1}, n \in N, n \leq 4\}}_{\text{Set builder form.}}$

in roster form

Answer.

we have formula for values of x.

$$x = \frac{n}{n+1}$$

Here n is a natural number ($\because n \in N$) and it is less than or equal to 4.

it means $n = 1, 2, 3, 4$

$x = \frac{n}{n+1} \longrightarrow n = 1 \Rightarrow x = \frac{1}{2}$

$n = 2 \Rightarrow x = 2/3$

$n = 3 \Rightarrow x = 3/4$

$n = 4 \Rightarrow x = 4/5$

values of $x = \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$.

$\therefore A = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\right\}$ = Roster form.

Exercise $\boxed{1.1}$

Empty set: Set which has no element.

$\phi = \{\ \} =$ empty/null/void set.

General name = phi (फाई)

e.g. Set of persons living on sun $= \{\ \} = \phi$

Note: do not write ϕ in bracket
like $\{\phi\}$ ← this is wrong.

(it mean there is an element ϕ in the set)

this is not empty set.

Singleton Set : which has exactly one element

example:

Set of even prime number $= \{2\}$

"2 is the only prime number which is even also."

Finite Set: having finite number of (Countable) elements.

Infinite set : having infinite (∞) number of elements.

examples:

$A = \{1, 2, 3, 4, 5\} \rightarrow$ Finite set (no. of elements = 5)

$B = \{1, 2, 3, 4, \ldots\ldots\} \rightarrow$ Infinite set (no. of elements $= \infty$)

Cardinal number of a set A = $n(A)$ = no. of elements in set A.

Example. $A = \{1,2,3\} \longrightarrow n(A) = 3$

$B = \{a, e, i, o, u\} \rightarrow n(B) = 5$

$\phi = \{\ \} \longrightarrow n(\phi) = 0$

$C = \{1, 2, 3, ----\} \longrightarrow n(c) = \infty$ = Not Defined

Equal Sets : Two sets A & B are said to be equal sets if there all elements are same and written as $A = B$.

Example.

$A = \{1,2,3\} = B = \{3, 2, 1\}$

$A = \{x, a\} = B = \{a, x\}$

But $A = \{1,2,3\}$, $B = \{a, b, c\} \Rightarrow A \neq B$

$A = \{1,2\}$, $B = \{1,2,3\} \Rightarrow A \neq B$

Extra

Equivalent Sets : two Sets A & B are said to be equivalent if their no. of elements are equal (~~form~~ their elements may or may not be same)
written as → $\boxed{A \sim B}$

e.g.

$A = \{1, 2, 3\}$, $B = \{a, b, c\}$ → $A \neq B$ but $A \sim B$

$A = \{1, 2, 3\}$, $B = \{1, 2\}$ → $A \neq B$ & $A \nsim B$

$A = \{1, 2, 3\}$, $B = \{1, 2, 3\}$ → $A = B$ & $A \sim B$

All equal sets are equivalent but all equivalent sets may not be equal.

Subsets :

X = Set of all students in your school.
Y = Set of all students in your class 11^{th}.

Clearly Y is smaller set and X is bigger set.
Students in Y are already in X but students in X (whole school) may or may not be in Y.

in this situation Y is sub set of X
& X is super set of Y.

For Example: class 11 is subset of School

class 11 $\subset$ School

Subset   Sign of subset.   Superset.

Definition of Subset:

a set 'A' is said to be a subset of a set B if each element of 'A' is also an element of 'B'. written as $A \subset B$

you can under this definition by replacing 'A' by class 11^{th}, 'B' by 'school'.

e.g. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A_1 = \{1, 2\} \longrightarrow A_1 \subset A$

$A_2 = \{2, 7, 8, 9, 10\} \longrightarrow A_2 \subset A$

$A_3 = \{7, 8, 15, 2\} \longrightarrow A_3 \not\subset A$

(A_3 is not subset of A)

$\because 15 \in A_3$

but $15 \notin A$

Note:

① $A \subset A$ (each set is subset of itself)

② $\phi \subset A$ (empty set is subset of every set)

③ For two equal sets A and B

$A = B$ so we can say that $A \subset B$ & $B \subset A$.

$\boxed{A = B \iff A \subset B \text{ and } B \subset A}$

(4) If A is subset of B but there are some elements in B which are not in A. then 'A' is proper subset of B (in other words $A \subset B$ but $A \neq B$)

Here B is super set of 'A'.

Other wise 'A' will be improper subset.

Example:

Let $B = \{1, 2, 3\}$

$\boxed{\text{Subsets of B}}$

$A_1 = \phi$
$A_2 = \{1\}$
$A_3 = \{2\}$
$A_4 = \{3\}$
$A_5 = \{1, 2\}$
$A_6 = \{2, 3\}$
$A_7 = \{3, 1\}$

Proper subsets of B.

$A_8 = \{1, 2, 3\}$ } improper subsets of B.

Because $\underline{A_8 = B}$.

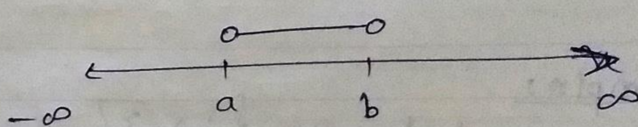
Intervals as subsets of 'R'

Real numbers occur continuously on number line.

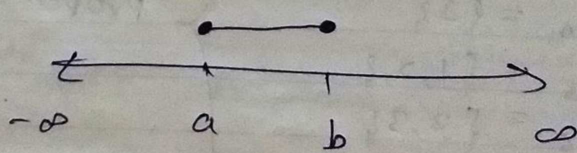
we use intervals to understand inequality.

smaller no. — larger — smaller — larger

If $a < x < b$, then $x \in (a, b)$



If $a \le x \le b$, then $x \in [a, b]$



Note. Understant I^{st} case

x is not equal to a & b, x is just between them. In the form of intervals we write $x \in (a, b)$. Here we use small bracket.

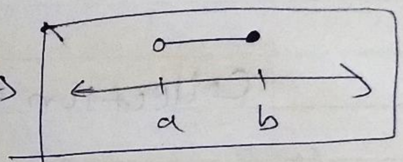
For number line we use like o——o (both side Hollow)

understand II case:

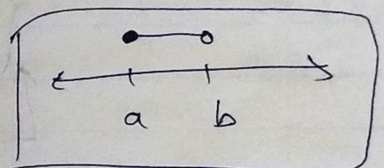
x is equal to a & b and also 'x' lies between them. So we use square bracket. For number line, we use •—•, both side filled.

Let's do more cases:

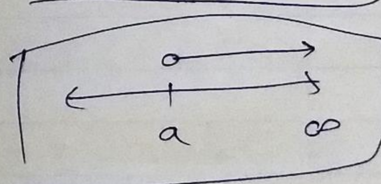
$\boxed{a < x \le b} \Rightarrow \boxed{x \in (a, b]} \Rightarrow$



$\boxed{a \le x < b} \Rightarrow \boxed{x \in [a, b)} \Rightarrow$



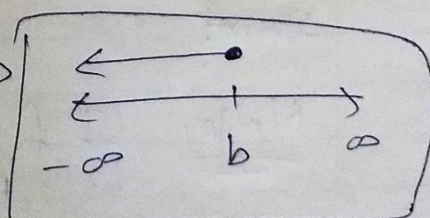
$\boxed{a < x} \Rightarrow \boxed{x \in (a, \infty)} \Rightarrow$



$a < x < \infty$

Because each number is less than '∞'

$\boxed{x \le b} \Rightarrow x \in (-\infty, b] \Rightarrow$



$\boxed{-\infty < x \le b}$

because each number is great than $(-\infty)$

Also we can not include (∞) or $(-\infty)$ because they are not defined.

(P(A)) **Power set:** is the collection (set) of sub sets of any set.

Example: $A = \{1, 2\}$

Subsets of $A = \phi, \{1\}, \{2\}, \{1,2\}$

Collection of Subsets of A

$$= \{\phi, \{1\}, \{2\}, \{1,2\}\} = P(A)$$

Apply extra curly bracket (Beause inner sets are just ~~set~~ elements of power set)

~~Note~~

Note, If number of elements in A is m then number of subsets $= 2^m$.

So If $n(A) = m$, then $n(P(A)) = 2^m$.

power set = set of subsets

Universal set: (U) → (ξ)

A set which has all elements discussed in any problem.

For example
If in a problem there are A, B & C sets.

let $A = \{1, 2\}$, $B = \{1, 2, 3\}$, $C = \{7, 8\}$

Someone can assume $U = \{1, 2, 3, 7, 8\}$ ← It has all element of A, B, C.

universal set

Someone can assume $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Someone can assume $U = \{1, 2, 3, ----\} = N$
(Natural no.)

Venn Diagram: Understand sets using diagrams.

Generally Universal set is rectangle and nothing goes beyond this rectangle.

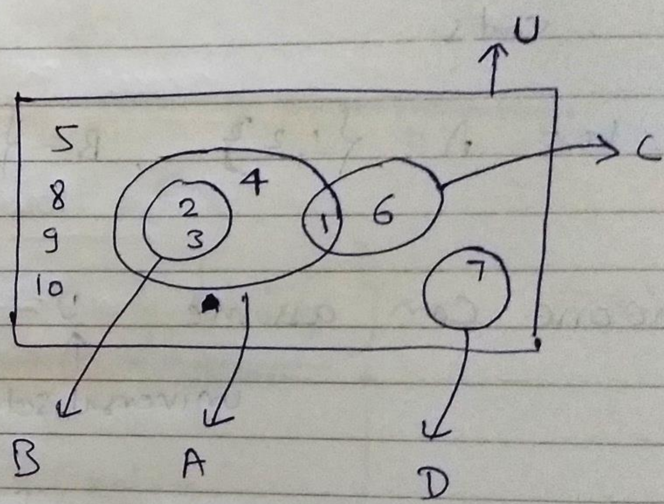
All other sets are Circles.

For example:

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 4\}$ $B = \{2, 3\}$ $C = \{1, 6\}$

$D = \{7\}$



5, 8, 9, 10 does not belong to any set but U.

Operations on sets:

1. Union of sets
2. intersection of sets
3. Complement of sets
4. Difference of sets

each operation gives new set.

① Union of Sets $\{A \cup B \Rightarrow A \text{ union } B\}$

$A \cup B$ is the set containing elements which belong to either A or B or both.

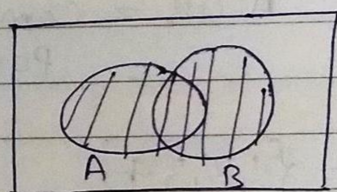
For examples:

$A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$

$\therefore A \cup B = \{1, 2, 3, 4, 5\}$

By Venn Diagram $\longrightarrow$

$A \cup B \longrightarrow$



$\rightarrow A \cup B$

So $A \cup B = \{x : \underline{x \in A} \text{ or } \underline{x \in B}\}$

↑

notice that we used word 'or'

Properties

① $A \cup A = A$

② $A \cup U = U$ = universal set.
↑ universal set

③ $A \cup \phi = A$

④ $A \cup B = B \cup A \rightarrow$ (Commutative)

⑤ $(A \cup B) \cup C = A \cup (B \cup C) \longrightarrow$ (Associative property)

(2) Intersection of Sets:

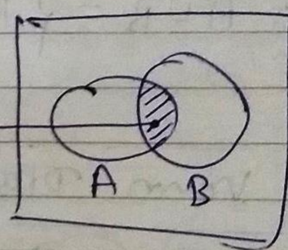
{$A \cap B \Rightarrow$ A intersection B} = set of Common elements.

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

notice the word 'and'

By Venn Diagram

$A \cap B$ = Common Portion



e.g. $A = \{1, 2, 3, 4\}$
$B = \{1, 3, 7, 8, 9\}$

$A \cap B = \{1, 3\} \Rightarrow$ Because 1 & 3 are Common in Both A and B

Properties:

$$\begin{cases} A \cap A = A \\ A \cap U = A \\ A \cap \phi = \phi \end{cases}$$

$A \cap B = B \cap A$ Commutative

$(A \cap B) \cap C = A \cap (B \cap C)$ Associative

Distributive Law:

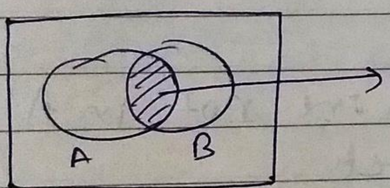
(1) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

Understand it like $(A+B) \times C = A \times C + B \times C$

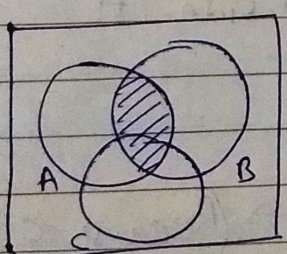
(2) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

Understand it like $(a+b) \times c = a.c + b.c$

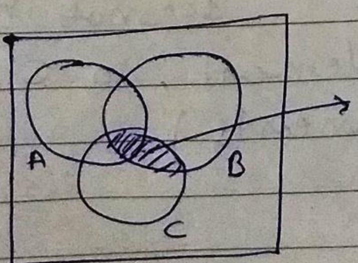
Examples



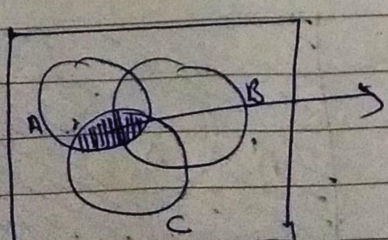
$A \cap B$



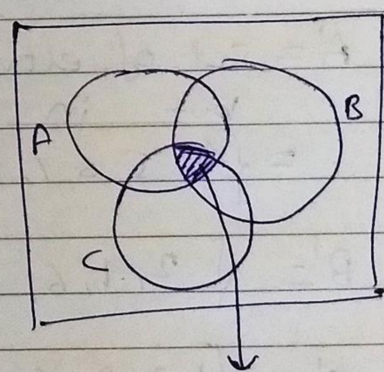
$A \cap B$



$B \cap C$

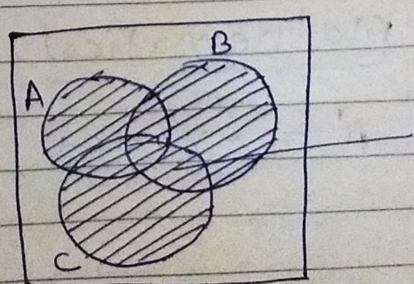


$A \cap C$



$A \cap B \cap C$

(Common portion of all)



$A \cup B \cup C$

Date / /

(3) Difference of sets: A − B ⇒ A minus B

A − B = Set of elements of only A which does not belong to B

For example.

$A = \{1, 2, 3, 4, 5\}$ $B = \{3, 4, 5, 6, 7\}$

$\Rightarrow A - B = \{1, 2, 3, 4, 5\} - \{3, 4, 5, 6, 7\}$

Here 3, 4, 5 also lies in 'B' so we remove them.

$\Rightarrow A - B = \{1, 2\}$ we write elements of only 'A' forget about remaining elements of B, they will not come in A-B.

Now $B - A = \{\underline{3, 4, 5}, \underbrace{6, 7}_{\text{come}}\} - \{\underbrace{1, 2}_{\text{forget.}}, \underline{3, 4, 5}\}$

$= \{6, 7\}$

One more Example.

$A = \{1, 2, 3\}$ $B = \{2, 3\}$

$A - B = \{1, \underline{2, 3}\} - \{\underline{2, 3}\} = \{1\}$

Come

But $B - A = \{\underline{2, 3}\} - \{1, \underline{2, 3}\} = \phi = \{\ \}$

Every element is removed

(4) Complement of a Set 'A' = $\underbrace{A' = A^c = \bar{A}}_{\text{representation}}$

Definition = set of elements which are not in A

example. Let $U = \{1, 2, 3, \ldots, 9, 10\}$

$A = \{1, 2, 3\}$

$B = \{1, 3, 5, 7, 9\}$

$C = \{\ \} = \phi$

A' = set of elements which are not in 'A' but in universal set

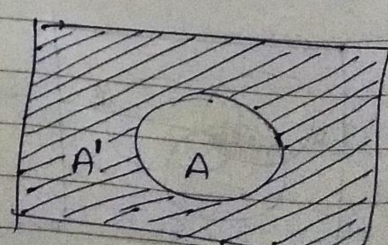
$= \{4, 5, 6, 7, 8, 9, 10\}$ ← outside A.

$B' = \{2, 4, 6, 8, 10\}$

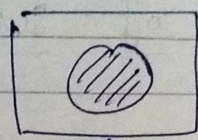
$C' = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ → (Because 'C' doesnot have any element, so C' has all elements)

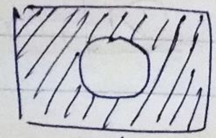
mathematical Definition:

$A' = \{x : x \in U, x \notin A\}$



Properties :


A


A'

① $(A')' = A$

② $A \cup A' = U$ = universal set

↑ union

③ $A \cap A' = \phi$ = empty set

④ $U' = \phi$ & $\phi' = U$

⑤ **De Morgan's Law:**

$(A \cup B)' = A' \cap B'$

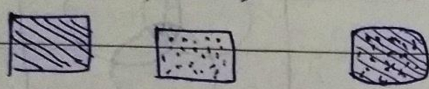
Sign change.

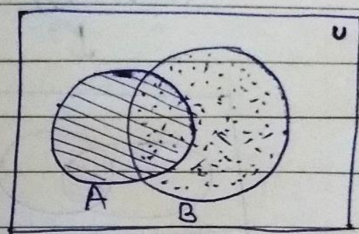
$(A \cap B)' = A' \cup B'$

Practical problems on Sets:

(1)

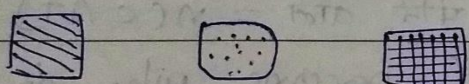
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

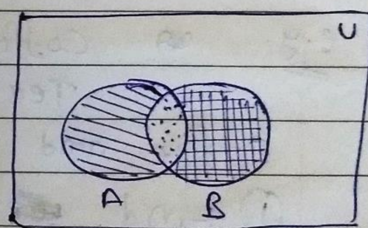




(2)

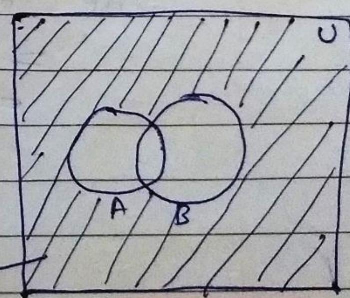
$n(A \cup B) = n(A-B) + n(A \cap B) + n(B-A)$





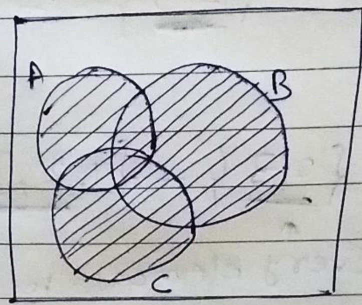
(3)

$n(\overline{A \cup B}) = n(U) - n(A \cup B)$



$\overline{A \cup B}$

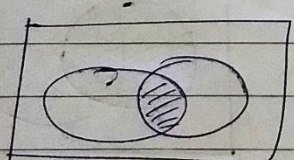
(4) $n(A \cup B \cup C) = n(A) + n(B) + n(C)$
$- n(A \cap B) - n(B \cap C) - n(C \cap A)$
$+ n(A \cap B \cap C)$



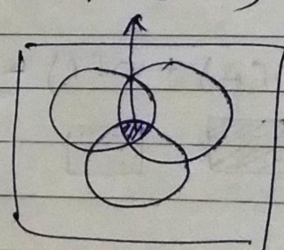
Here ▨ → $A \cup B \cup C$.

<u>Note</u>: to solve practical problems on sets, always start with the most common portion.

$n(A \cap B)$



$n(A \cap B \cap C)$



<u>e.g.</u> Coffee पीने वाले $= n(C) = 15$
Tea पीने वाले $= n(T) = 10$
Coffee and Tea पीने वाले $= n(C \cap T) = 6$ } Let

(1) Find no. of persons who drink at least one of coffee & tea

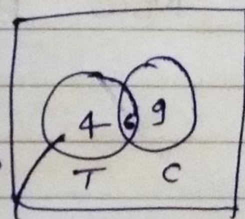
$= n(C \cup T) = n(C) + n(T) - n(C \cap T)$

$\Rightarrow n(C \cup T) = 15 + 10 - 6$

$= 19$

② Find who drink only Tea not Coffee

1st method by Venn Diagram
(Start by labelling 6 in $C \cap T$)



Ans = 4

2nd method

we know that

$$n(A) = n(A-B) + n(A \cap B)$$

$$\Rightarrow n(T) = n(T-C) + n(C \cap T)$$

$$\Rightarrow 10 = n(T-C) + 6$$

$$\Rightarrow n(T-C) = 4$$ = Tea only not coffee.